Description of M-branes with Lie 3-algebra

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What is M-brane?

- ½-BPS solution of 11-dim SUGRA (in low energy limit)
  - near horizon geometry: $\text{AdS}_{4,7} \times S^{7,4}$ spacetime
  - d.o.f.: $N^{3/2}$ (for N M2-branes) or $N^3$ (for N M5-branes)

- CFT on M-branes? (using AdS/CFT correspondence)
  - The gauge symmetry is not defined by Lie algebra...?
  - Of course, this CFT must have 16 supersymmetries.
  - There are no parameters, when the background is flat.

Therefore, possible formulations seem very restrictive!
New M-brane models

- Multiple M2-branes
  - BLG theory [Bagger-Lambert ’07] [Gustavsson ’07]
    3-dim N=8 SUSY, gauge sym. with Lie 3-algebra
  - ABJM theory [Aharony-Bergman-Jafferis-Maldacena ’08]
    3-dim N=6 SUSY, gauge sym. with Lie algebra

- Multiple M5-branes
  - Nonabelian (2,0) theory [Lambert-Papageorgakis ’10]
    6-dim N=(2,0) SUSY, gauge sym. with Lie 3-algebra
Our plan of research is...

to see if these theories can reproduce properly S, T, U-duality relation among M-branes and the branes in type IIA/IIB superstring theory.

If they do in some way, we can regard it as a piece of evidence to show that these theories describe M-brane system!

For BLG theory (M2-branes) : [Ho-Matsuo-SS ’09] [Kobo-Matsuo-SS ’09]
For nonabelian (2,0) theory (M5-branes) : [Honma-Ogawa-SS ’11]
U-duality network

- M2-brane
- M5-brane
- NS5-brane
- KK monopole
- D2-brane
- D4-brane
- D6-brane
- D1-brane
- D3-brane
- D5-brane
- D7-brane

Type IIA

Type IIB
Our strategy is...

to consider Lie 3-algebra including Lie algebra, since gauge sym. of D-brane system is defined by Lie algebra U(N).

General definition: \[ [T^a, T^b, T^c] = f^{abc}_d T^d \]

Example we adopt: \[ [u, T^i, T^j] = i f^{ij}_k T^k \]

In order to define gauge sym., Lie 3-algebra must satisfy some conditions, which require additional generators. However, our fundamental idea is this.
Lie 3-algebra vs. Compactification

- **Compactification**

  We assign a **VEV** to $u$-component field:

  \[
  [X^I, X^J, X^K] \rightarrow \lambda^I [X^J, X^K]
  \]

  This can be regarded as a **coupling constant** or a **compactification radius**.

- **We should choose VEV’s without breaking gauge sym. nor supersymmetry to obtain Dp-brane theory.**

- **We assume that the fluctuation around a VEV is considerably smaller than other fluctuations.**
Lie 3-algebra vs. T-duality

T-duality : Taylor’s T-duality  [Taylor ’96]

We choose a kind of loop algebra as Lie algebra:

\[
[u_0, T^{i}_{\bar{m}}, T^{j}_{\bar{n}}] = i f^{i j}_{\quad k} T^{k}_{\bar{m}+\bar{n}}
\]
\[
[u_0, u_a, T^{i}_{\bar{m}}] = m_a T^{i}_{\bar{m}}
\]

Similarly, this means the compactification on a torus.

And we regard \( m_a \) as KK momentum on a torus \( T^d \):

\[
X^{I}_{i}(x, y) = \sum_{\bar{m}} X^{I}_{(i\bar{m})} e^{-i\bar{m} \cdot \bar{y}}
\]

This means the extension of worldvolume for the torus directions by correcting T-component fields.
Our results at this stage

- These theories with Lie 3-algebra seem to successfully reproduce U-duality relation.

- Almost all parameters can be recovered, but unfortunately, some are lacked at this stage.

- However, at least, there are no contradictions with known facts.

Now let me explain the detail of our discussion...
Contents

1. Introduction  (finished)
2. BLG theory  (for M2-branes)
3. Nonabelian (2,0) theory  (for M5-branes)
4. Conclusion
BLG theory

- M5-brane
- M2-branes
- NS-NS 2-form field
- R-R (p-3)-form fields
- graviton
- Dp-branes on (p-2)-dim torus
- NS-NS 2-form field
- R-R (p-5)-form fields
- F1-strings
**BLG theory**

Field contents of M2-brane theory

\[ X^I_a : \text{transverse scalars} \quad (8 \text{ d.o.f.)} \quad I = 3, \ldots, 10 \]

\[ \Psi_\mu : \text{superpartner fermion} \quad (8 \text{ d.o.f.)} \quad \mu = 0, 1, 2 \]

\[ A_{\mu \alpha \beta} : \text{Chern-Simons gauge field} \quad (0 \text{ d.o.f.)} \]

The coordinates for worldvolume directions are fixed by static gauge:

\[ X^\mu (\sigma) = \sigma^\mu 1 \]

This corresponds to the center-of-mass mode which is decoupled from the theory.
\[ S = \int_{\mathcal{M}} d^3x \ L = \int_{\mathcal{M}} d^3x \left( L_X + L_\Psi + L_{int} + L_{pot} + L_{CS} \right) \]

\[ L_X = -\frac{1}{2} \left\langle D_\mu X^I, D^\mu X^I \right\rangle \]

\[ L_\Psi = \frac{i}{2} \left\langle \bar{\Psi}, \Gamma^\mu D_\mu \Psi \right\rangle \]

\[ L_{int} = \frac{i}{4} \left\langle \bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi] \right\rangle \]

\[ L_{pot} = -\frac{1}{12} \left\langle [X^I, X^J, X^K], [X^I, X^J, X^K] \right\rangle \]

\[ L_{CS} = \frac{1}{2} f^{abcd} A_{ab} \wedge dA_{cd} + \frac{i}{3} f^{cd_a} g f^{e f g b} A_{ab} \wedge A_{cd} \wedge A_{e f} \]

where the covariant derivative \( D_\mu \Phi_a = \partial_\mu \Phi_a - f_{cdb}^a A_{\mu cd} \Phi_b \)
On Lie 3-algebra

Fundamental identity (∼ Jacobi identity)

\[ [T^a, T^b, [T^c, T^d, T^e]] = [[T^a, T^b, T^c], T^d, T^e] \]
\[ + [T^c, [T^a, T^b, T^d], T^e] + [T^c, T^d, [T^a, T^b, T^e]] \]

Condition for inner product

\[ \langle [T^a, T^b, T^c], T^d \rangle + \langle T^c, [T^a, T^b, T^d] \rangle = 0 \]

Then additional element \( v \) must be introduced:

\[ [v, \star, \star] = 0 \quad \text{and} \quad \langle v, v \rangle = 1 \]

\( v \) is not equal to 1!

Instead, we set \( \langle u, v \rangle = 1 \) whose metric becomes Lorentzian, which means the theory has ghosts...!?
New kind of Higgs mechanism

To eliminate the **ghosts**, we put VEV’s for them without breaking gauge sym. nor supersymmetry.

From systematic discussion, we can in fact do this, when we choose a kind of Lie 3-algebra:

\[
[u_0, u_a, u_b] = 0
\]

\[
[u_0, u_a, T^i_{m\bar{m}}] = m_a T^i_{m\bar{m}}
\]

\[
[u_0, T^i_{m\bar{m}}, T^j_{\bar{n}\bar{n}}] = i f^{ijk} T^k_{m\bar{m}+\bar{n}} + m_a v^a \delta_{m\bar{m}+\bar{n}} \delta^{ij}
\]

\[
[T^i_{m\bar{m}}, T^j_{\bar{n}\bar{n}}, T^k_l] = -i f^{ijk} v^0 \delta_{m\bar{m}+\bar{n}+\bar{l}}
\]

Note: \(u\) is not produced by commutators, and \(v\) is center.
**M2 to D2**

Lie 3-algebra: \[ \{ T^i, u, v \} \]

VEV: \[ X^I_u = \lambda \delta^I_{10}, \text{ otherwise } = 0. \]

Result: 3-dim super Yang-Mills action

\[
\mathcal{L} = -\frac{1}{2} (\hat{D}_\mu \hat{X}^I)^2 + \frac{i}{4} \bar{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} - \frac{1}{4\lambda^2} \hat{F}_{\mu\nu}^2 \\
+ \frac{i}{2} \bar{\Psi} \Gamma^I [\hat{X}^I, \hat{\Psi}] + \frac{\lambda^2}{4} [\hat{X}^I, \hat{X}^J]^2
\]

where \( I = 3, \cdots, 9 \). Note that Chern-Simons field becomes Yang-Mills field by eating 1 d.o.f. of \( X^{10} \).
Lie 3-algebra: \( \{T_{m_i}, u_a, v^a\} \quad (a = 0, \cdots, p - 2) \)

VEV's: \( X^I_{u_a} = \lambda^I a, \) otherwise = 0.

Result: super Yang-Mills action on a torus \( T^{p-2} \)

\[
L = \int \frac{d^d y}{(2\pi)^d} \sqrt{g} (L_A + L_{FF} + L_X + L_\Psi + L_{pot} + L_{int} + L_{td})
\]

\[
L_A = -\frac{1}{4G^{00}} \left( \tilde{F}_{\mu \nu}^2 + 2g^{ab}\tilde{F}_{\mu a}\tilde{F}_{\nu b} + g^{ac}g^{bd}\tilde{F}_{ab}\tilde{F}_{cd} \right), \quad L_{FF} = -\frac{G^{00}}{8G^{00}} \left( 4\epsilon^{\mu \nu \lambda} \tilde{F}_{\mu a} \tilde{F}_{\nu \lambda} \right)
\]

\[
L_X = -\frac{1}{2} \left( \tilde{D}_{\mu} \tilde{X}^I P^{IJ} \tilde{D}_{\mu} \tilde{X}^J + g^{ab}\tilde{D}_a \tilde{X}^I P^{IJ} \tilde{D}_b \tilde{X}^J \right), \quad L_\Psi = \frac{i}{2} \tilde{\Psi} \left( \Gamma^\mu \tilde{D}_\mu + \Gamma^a \tilde{D}_a \right) \tilde{\Psi}
\]

\[
L_{pot} = -\frac{G^{00}}{4} [P^{IK} \tilde{X}^K, P^{JL} \tilde{X}^L]^2, \quad L_{int} = \frac{i\sqrt{G^{00}}}{2} \tilde{\Psi} \Gamma_I [P^{IJ} \tilde{X}^J, \tilde{\Psi}]
\]

where \( P \) is a projector onto the subspace perpendicular to torus, \( G \) and \( g \) are defined in terms of VEV's.
**U-duality for M2 and D3-branes**

- **S-duality**: string coupling can be read from Yang-Mills coupling as \( g_s^{-1} = \tau_2 \).

- **T-transformation**: value of axion can be read from FF term as \( C_0 = \tau_1 \).

- **Taylor’s T-duality**: D3-branes or D2-branes with open string modes

Thus **U-duality group** \( SL(2, \mathbb{Z}) \rtimes \mathbb{Z}_2 \) is reproduced!

**T-duality relation**: compactification radius can be read from metric on circle. This corresponds properly to length of \( \tilde{\lambda}^1 \) under T-duality.
**U-duality for M2 and Dp-branes**

\[ d = p - 2 \]

**Parameters**
- string coupling
- metric on torus
- R-R (d-1)-form field
- NS-NS B-field
- Taylor’s T-duality
- interchange of directions

We can write them in terms of VEV’s which are transformed under the \( \text{SL}(d+1,\mathbb{Z}) \) transformation:

\[
\tilde{x}^a' = \Lambda^{a}_{b} \tilde{x}^b
\]

They are related to setting of Lie 3-algebra which realizes \( \text{SO}(d,d;\mathbb{Z}) \) transformation.

**U-duality group** is \( E_{d+1}(\mathbb{Z}) = \text{SL}(d + 1, \mathbb{Z}) \rtimes SO(d, d; \mathbb{Z}) \).

Some R-R fields are lacked in our setup, regrettably.
Two M2-branes

A₄ algebra : \[ [T^a, T^b, T^c] = \epsilon^{abcd} T^d \]

In fact, this is the only example of positive definite Lie 3-algebra!

An M5-brane (as infinite number of M2-branes)

NP bracket : \[ [T^a, T^b, T^c] = \epsilon^{\mu \nu \rho} \partial_{\mu} T^a \partial_{\nu} T^b \partial_{\rho} T^c \]

This is defined on 3-dim Nambu-Poisson manifold. \( \mu = 1, 2, 3 \)

In fact, BLG theory with Nambu-Poisson bracket describes an M5-brane in C₃-field background.
Nonabelian (2,0) theory

- M5-branes
- NS5-branes
- R-R (p-5)-form fields
- graviton
- dilaton
- Dp-branes on (p-4)-dim torus
- NS-NS 2-form field
- R-R (p-7)-form fields
Nonabelian (2,0) theory

Field contents of M5-brane theory

\( X^I_a \): transverse scalars (5 d.o.f.)
\( \Psi_a \): fermion (8 d.o.f.)
\( B_{\mu \nu, a} \): self-dual 2-form field (3 d.o.f.)
\( A_{\mu ab} \): gauge field
\( C^\mu_a \): auxiliary field (0 d.o.f.)

The gauge fixing of coordinates for worldvolume directions seem to be relaxed as

\[
X^\mu(\sigma) = \sigma^\mu 1 + C^\mu_a(\sigma) T^a \quad (l_p = 1)
\]
Equations of motion

\[ D_{\mu}^2 X^{I}_{a} - \frac{i}{2} [\epsilon^\mu_{\lambda \nu \rho \sigma}, \Gamma^\lambda_{\mu \nu} \Gamma^{I}_{\lambda \nu} \Psi]_{a} - [\epsilon^\mu_{\lambda \nu \rho \sigma}, X^{I}, [\epsilon^\mu_{\lambda \nu \rho \sigma}, X^{I}, X^{I}]]_{a} = 0 \]

\[ \Gamma^\mu_{\nu} D_{\mu} \Psi_{a} + \Gamma^\mu_{\nu} \Gamma^{I}_{\lambda \nu} [\epsilon^\mu_{\lambda \nu \rho \sigma}, X^{I}, \Psi]_{a} = 0 \]

\[ D_{[\mu} H_{\nu \rho \sigma]} + \frac{1}{4} \epsilon_{\mu \nu \rho \sigma \lambda \tau} [\epsilon^\lambda_{\mu \nu \rho \sigma}, D^\tau X^{I}]_{a} + \frac{i}{8} \epsilon_{\mu \nu \rho \sigma \lambda \tau} [\epsilon^\lambda_{\mu \nu \rho \sigma}, \Psi, \Gamma^\tau \Psi]_{a} = 0 \]

\[ \tilde{F}_{\mu \nu}^{\rho} - C_{\rho}^{\mu} H_{\mu \nu \rho}, d f_{c d b}^{c d b} = 0 \]

\[ D_{\mu} C_{\nu}^{\mu} = 0 \]

Constraints:

\[ C_{c}^{\mu} D_{\mu} X^{I}_{d} f_{c d b}^{c d b} = C_{c}^{\mu} D_{\mu} \Psi_{d} f_{c d b}^{c d b} = C_{c}^{\mu} D_{\mu} H_{\nu \rho \sigma}, d f_{c d b}^{c d b} = C_{c}^{\mu} C_{c}^{\nu} f_{c d b}^{c d b} = 0 \]

Covariant derivative:

\[ D_{\mu} \Phi_{a} = \partial_{\mu} \Phi_{a} - f_{c d b}^{c d b} A_{\mu c d} \Phi_{b} \]
Existence of B-field

In this theory, only its field strength appears.

\[ H_{\mu\nu\rho,a} = D_\mu B_{\nu\rho,a} + D_\nu B_{\rho\mu,a} + D_\rho B_{\mu\nu,a} \]

\[ \tilde{F}_{\mu\nu}^{\ b}_a = \partial_\nu \tilde{A}_\mu^{\ ab} - \partial_\mu \tilde{A}_\nu^{\ ab} - \tilde{A}_\mu^{\ ac} \tilde{A}_\nu^{\ cb} + \tilde{A}_\nu^{\ ac} \tilde{A}_\mu^{\ cb} \]

At this stage, B-field cannot be defined properly, since nonlinear terms don’t correspond to each other.

Free part of Lagrangian

\[ L = -\frac{1}{2} (D_\mu X^I)^2 + \frac{i}{2} \bar{\Psi} \Gamma^\mu D_\mu \Psi - \frac{1}{12} H_{\mu\nu\rho}^2 + \cdots \]
M5 to D4

Lie 3-algebra: \( \{ T^i, u, v \} \)

VEV: \( C_\mu^u = \lambda \delta_5^\mu \), otherwise = 0.

Result: 5-dim super Yang-Mills action

\[
\mathcal{L} = -\frac{1}{2} (\hat{D}_\mu \hat{X}^I)^2 + \frac{i}{4} \bar{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} - \frac{1}{4 \lambda^2} \hat{F}_{\mu \nu}^2 \\
+ \frac{i}{2} \bar{\Psi} \Gamma^I [\hat{X}^I, \hat{\Psi}] + \frac{\lambda^2}{4} [\hat{X}^I, \hat{X}^J]^2
\]

can reproduce the resultant equations of motion, where \( \mu = 0, \cdots, 4 \) and \( \hat{F}_{\mu \nu}^j i = \lambda H_{\mu \nu 5,k} f^{k,j} i \).
M5 to Dp

Lie 3-algebra: \( \{ T^i_{\hat{m}}, u_0, u_a, v^0, v^a \} \) \((a = 1, \cdots, p - 4)\)

VEV’s: \( C_{u_0}^\mu = \lambda \delta_5^\mu, \; X_{u_a}^I = \lambda^{Ia}, \) otherwise = 0.

Result: super Yang-Mills action on a torus \( T^{p-4} \)

\[
S = \lambda \int d^5x \frac{d^d y}{(2\pi)^d} \sqrt{g} L
\]

\[
L = -\frac{1}{2} (\hat{D}_\mu \hat{X}^I) P^{IJ} (\hat{D}_\nu \hat{X}^J) + \frac{i}{2} \hat{\bar{\Psi}} \Gamma_\mu \hat{D}_\mu \hat{\Psi} - \frac{1}{4\lambda^2} \hat{F}_{\mu\nu}^{2}
\]

\[
-\frac{\lambda^2}{4} [P^{IK} \hat{X}^K, P^{JL} \hat{X}^L]^2 + \frac{i\lambda}{2} \hat{\bar{\Psi}} \Gamma_I [P^{IJ} \hat{X}^J, \hat{\Psi}]
\]

where the metric on torus is defined in terms of VEV’s.
Comments on M5 to Dp

Generalization of VEV’s

\[ C^\mu_{u_0} = \lambda \delta^\mu_5, \quad C^\mu_{u_a} = \tilde{\lambda}^a \delta^\mu_5, \quad X^I_{u_a} = \lambda^{Ia}. \]

However, equations of motion remain unchanged.

Total derivative terms?

This may be related to T-transformation.

In fact, the term \( \epsilon^{\mu\nu\rho\sigma\lambda\tau} F_{\mu\nu}{}^a_b F_{\rho\sigma}{}^b_c F_{\lambda\tau}{}^c_a \) in the original theory gives the Chern-Simons term in the resultant theory:

\[ L \supset \lambda^2 \tilde{\lambda}^a \epsilon^{\mu\nu\rho\sigma\lambda\tau} \hat{F}_{\mu\nu,i} \hat{F}_{\rho\sigma,j} \hat{F}_{\lambda\tau,k} f^{il} f^{jm} f^{kn} f^{kl} + \cdots \]

(This discussion is meaningful, since B-field never appear.)
M5 to IIA NS5

Lie 3-algebra: \( \{T^i, u, v\} \)

VEV: \( X^I_u = \lambda \delta^I_{10}, \) otherwise \( = 0. \)

Result: practically simple copies of free theory

\[
\begin{align*}
\partial_\mu^2 X^i_a - \lambda^2 [C^\mu, [C_\mu, X^i]]_a &= 0 \\
\partial_\mu^2 X^{10}_a &= 0 \\
\Gamma^\mu \partial_\mu \Psi_a - \lambda \Gamma_\mu \Gamma^{10} [C^\mu, \Psi]_a &= 0 \\
\partial_{[\mu} H_{\nu\rho\sigma]}_a - \frac{\lambda}{4} \epsilon_{\mu\nu\rho\sigma\lambda \tau} [C^\lambda, (\partial^\tau X^{10} + \lambda \tilde{A}^{\tau 0})]_a &= 0 \\
\tilde{F}_{\mu\nu}^0_a - [C^\rho, H_{\mu\nu\rho}]_a &= 0 \\
(\nu = 6, \cdots, 9)
\end{align*}
\]
M5 to IIB NS5

Lie 3-algebra: \( \{ T^i_{m}, u_a, v^a \} \) (\( a = 0, 1 \))

VEV's: \( X^I_{u_0} = \lambda^0 \delta^I_{10}, \ C^\mu_{u_1} = \lambda^1 \delta^\mu_{5} \).

Result: practically simple copies of free theory of 6-dim N=(1,1) super Yang-Mills action

\[
L = -\frac{1}{2}(\hat{D}_\mu \hat{X}^i)^2 + \frac{i}{2} \hat{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} - \frac{1}{4(\lambda^1)^2} \hat{F}_{\mu\nu}^2 + \cdots
\]

where \( \underline{\mu} = 0, \cdots, 4, y \). The covariant derivatives are practically the partial derivatives. It is interesting that Yang-Mills field comes from \( A_{\mu u_1 (i \tilde{m})} \) in this case.
M5 to IIA KK monopole

- Lie 3-algebra: \( \{ T^i_{\bar{m}}, u^a, v^a \} \) \( (a = 0, 1, 2) \)

- VEV’s: \( X^I_{u_0} = \lambda^0 \delta^I_{10} \), \( C^\mu_{u_1} = \lambda^1 \delta^\mu_5 \), \( X^I_{u_2} = \lambda^2 \delta^I_9 \).

- Result: practically simple copies of free theory

It is almost the same as IIB NS5-brane case, except

\[
\hat{D}_\alpha \hat{X}^9 + \hat{D}_{y_1} \hat{X}^9 - (\lambda^0)^2 \lambda^1 \lambda^2 \hat{D}_{y_1} \partial_{y_2} \hat{C}^5 = 0
\]

From the viewpoint of Lorentz invariance for \( C^\mu \), it is natural to set it to zero by putting VEV’s for \( C^5 \).

The \( y_2 \) direction corresponds to Taub-NUT direction.

\( (\alpha = 0, \ldots, 4) \)
M5 to IIB KK monopole

- Lie 3-algebra: \( \{ T^i_{\hat{m}}, u^a, v^a \} \) (\( a = 0, 1 \))

- VEV’s: \( X^I_{u_0} = \lambda^0 \delta^I_{10} \), \( X^I_{u_1} = \lambda^1 \delta^I_9 \).

- Result: practically simple copies of free theory

It is almost the same as IIA NS5-brane case, except

\[
\partial^2_\mu \hat{X}^9 - (\lambda^0)^2 [\hat{C}_\mu, [\hat{C}_\mu, \hat{X}^9]] + i(\lambda^0)^2 \lambda^1 [\hat{C}_\mu, \partial_y \hat{C}^\mu] = 0
\]

Similarly, we set it to zero by putting VEV’s for \( C^\mu \). The y direction corresponds to Taub-NUT direction.

- S-self-duality: trivially satisfied (because of free theory)
**U-duality for M5, D5 and NS5**

- **S-duality**: string coupling for D5 is \( g_s^{-1} = \tau_2 |e|^3 / l_p^3 \).
  That for NS5 is related by \( \tilde{x}^0 \leftrightarrow \tilde{x}^1 \).

- **T-transformation**: value of axion can be read from FFF term as \( C_0 = \tau_1 |e|^3 / l_p^3 \).

- **Taylor’s T-duality**: D5-branes or D4-branes with open string modes

Thus **U-duality group** \( SL(2, \mathbb{Z}) \rtimes \mathbb{Z}_2 \) is reproduced!

- **T-duality relation** for compactified radius is also properly reproduced.
U-duality for M5 and Dp

\[ d = p - 4 \]

Parameters

- string coupling
- metric on torus
- R-R \((d-1)\)-form field
- NS-NS B-field
- Taylor’s T-duality
- interchange of directions

We can write them in terms of VEV’s which are transformed under the \(SL(d+1,\mathbb{Z})\) transformation:

\[ \tilde{\chi}^a = \Lambda^a_b \tilde{\chi}^b \]

They are related to setting of Lie 3-algebra which realizes \(SO(d,d;\mathbb{Z})\) transformation.

U-duality group is \(E_{d+1}(\mathbb{Z}) = SL(d+1,\mathbb{Z}) \rtimes SO(d,d;\mathbb{Z})\).

Some R-R fields are lacked in our setup, regrettably.
Comments

Two D4-branes...? (instead of M5-branes)

$A_4$ algebra: $[T^a, T^b, T^c] = \epsilon^{abcd} T^d$

In fact, expanding around a VEV, we obtain only 5-dim SU(2) super Yang-Mills.

An M9-brane...? (This may be a future work.)

NP bracket: $[T^a, T^b, T^c] = \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} \partial_{\dot{\mu}} T^a \partial_{\dot{\nu}} T^b \partial_{\dot{\rho}} T^c$

This is defined on 3-dim Nambu-Poisson bracket.

$\dot{\nu} = 1, 2, 3$
Conclusion
Towards the precise formulation...

- Lie 3-algebra enable us to introduce gauge sym. of M-brane worldvolume theory.

- Using new kind of Higgs mechanism, a part of U-duality relation can be reproduced correctly.

- This means that the theories in this seminar must be related to M-branes, but regrettably, they cannot reveal anything new about M-branes.

- To put the discussion forward, we should try new formulation, e.g. DBI-like one, with Lie 3-algebra.